ON CONVERGENCE TO EQUILIBRIUM OF INFINITE CLOSED JACKSON NETWORKS

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An Infinite Closed Jackson Network (ICJN; also known as Zero-Range process) on the infinite set of queues J is defined by its generator

$$A\,f(\eta) = \sum_{i\in J}\sum_{j\in J}\left[1_{\{\eta_i>0\}}\gamma_i p_{ij}\left(f(\ldots\eta_i-1\ldots\eta_j+1\ldots)-f(\eta)\right)\right],$$

where η_i is the number of customers in queue i, γ_i is the service rate at queue i, p_{ij} is the probability that customer leaving the queue i would join queue j and $f : J \to \mathbf{R}$ is a real-valued function measurable w.r.t. cylindric topology. Under conditions $\sup_i \gamma_i < \infty$, $\sup_i \sum_{j\in J}\gamma_j p_{ij} < \infty$ there exists a unique Feller process $\eta(t;\eta^{(0)})$ with generator A such that $\eta(0;\eta^{(0)}) = \eta^{(0)}$ a.s. [1].

ICJN with infinite number of customers ($\sum_i \eta_i^{(0)} = \infty$) is not ergodic because of continuum family of invariant measures, and there exists a number of sufficient conditions for devastating (overloading) of the queues with customers leaving to (coming from) infinite part of J [2,1]. The only example (dual to simple exclusion process) of convergence of $\eta(t;\eta^{(0)})$ to equilibrium measure was constructed in [3] for the case $J = \mathbf{Z} = \{0,\pm1,\pm2,\ldots\}$, $\gamma_i = 1$, $p_{i,i+1} = p_{i,i-1} = 1/2$ for all $i \in \mathbf{Z}$, with periodic initial deterministic configuration $\eta_i^{(0)} = \eta_{i+p}^{(0)}$. Then $\eta(t;\eta^{(0)}) \overset{w}{\to} G(\rho)$, where $G(\rho) = \prod_{i\in J} G_i(\rho)$, and $G_i(\rho)$ are distributions of geometric r.v. with expectation $EG_i(\rho) = \rho(\eta^{(0)}) = (\eta_1^{(0)} + \ldots + \eta_p^{(0)})/p$.

Here the new examples are constructed for IJCN on $J = \mathbf{N} = \{1,2,\ldots\}$.

Theorem. For any $M \in \mathbf{N}$ there exist (small) numbers $\delta_i > 0$, $i \in \mathbf{N}$ such that IJCN $\eta(t;\eta^{(0)})$ with $\gamma_i p_{i,i+1} = \gamma_{i+1}p_{i+1,i} = \delta_i$, $p_{i,i} = 1 - p_{i,i+1} - p_{i,i-1}$ for all $i \in J$ ($p_{1,0} \equiv 0$), converges weakly to $G(\rho)$ for any initial configurations $\eta^{(0)}$ bounded by M ($\eta_i^{(0)} \le M$ for all i) and having density $\rho(\eta^{(0)}) = \lim_{N\to\infty}(\eta_1^{(0)} + \ldots + \eta_N^{(0)})/N$: $\eta(t;\eta^{(0)}) \overset{w}{\to} G(\rho)$. If $\eta_i^{(0)} \le M$ for all i and limiting density $\rho(\eta^{(0)})$ is not defined, then $\eta_i(t;\eta^{(0)})$ is stochastically bounded for each $i \in \mathbf{N}$.

The proof uses variational metric and stochastical comparision of random processes and can be extended to non-neighbour transitions $p_{ij} \neq 0$, $|i-j| \ge 2$.

[1] D.V. Khmelev and E. Spodarev, *J. Math. Sci. (New York)* 106 (2001), no. 2, 2820-2829. [2] M.Ya.Kelbert, M.L.Konzevich, and A.N. Rybko. *Teor. ver. i yeyo prim.*, XXXII(2):379–382, 1988. [3] A. Galves and H. Guiol. *Markov Process. Related Fields*, 3(3):323–332, 1997.

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